

REIGATE GRAMMAR SCHOOL

FURTHER MATHEMATICS

Sixth Form Preparation 2020

Forename:

Surname:

Congratulations on choosing to study Maths in the Sixth Form. You have probably been told that there is a big jump between GCSE and A-Level, and this is certainly true of Mathematics. Many of the early topics build on the skills and techniques you have learnt at GCSE, so it is essential that you start your 6th form studies with a full knowledge of these. If you are not confident with the Mathematics in this workbook, then you may find yourself struggling from the start of the course.

We have prepared this workbook for you to complete over the holiday. All the questions in it are GCSE standard, so there is nothing you should not know how to do. If you complete it then you can assume that you know the basic ideas that you will need and we will know that you are committed to studying Mathematics.

You must show your working throughout this booklet to demonstrate to us the methods and techniques that you are using. **The only questions you may use a calculator on are marked with an image of a calculator**.



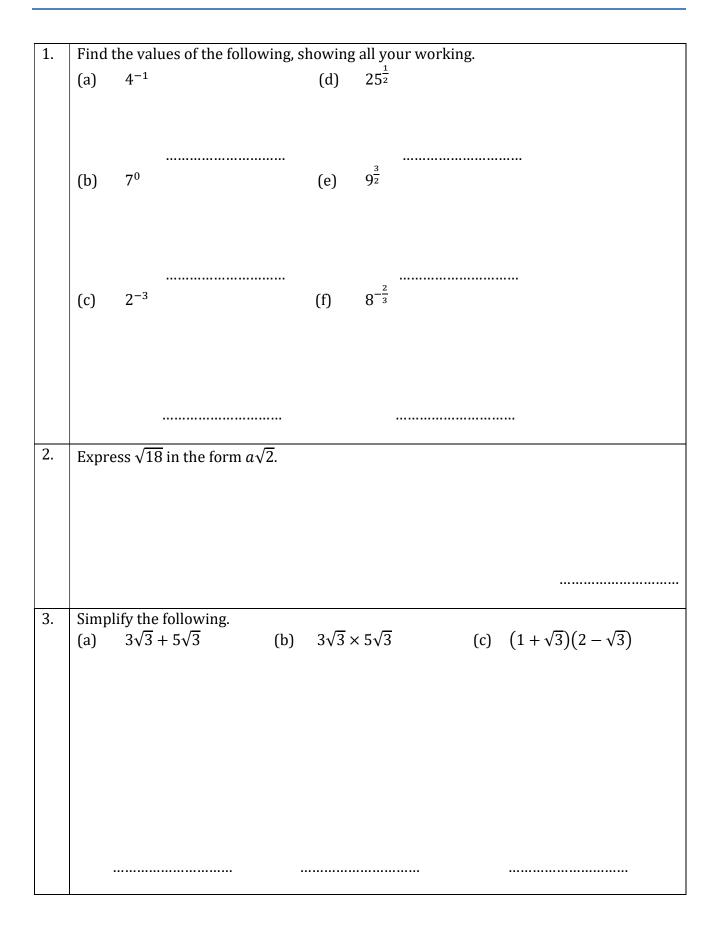
You should complete this workbook over the holiday and hand it in to a member of the Mathematics Department on the first day of term in September. If you do not hand it in, complete, then we may question whether or not you appear prepared to put in the efforts required to make a success of the course.

If you studied IGCSE, you should know how to complete all the questions in this workbook. If you did not study IGCSE, you may not know how to do the questions on Differentiation (Q38, 39 and 42) and on Functions (q32, 33, 34 and 37).

If you need any help on any questions you should first go to <u>www.hegartymaths.com</u> and use the videos online to help you. Failing that, you can email <u>gjs@reigategrammar.org</u> and you should get a reply within a few days.

You will have a test on the material you are expected to know near the beginning of the term, but don't worry, we will let you know exactly when in your first lessons. If you know everything in this workbook, you will know everything for the test, but if you don't then you might struggle. We will expect you to know most if not all the material in this workbook.

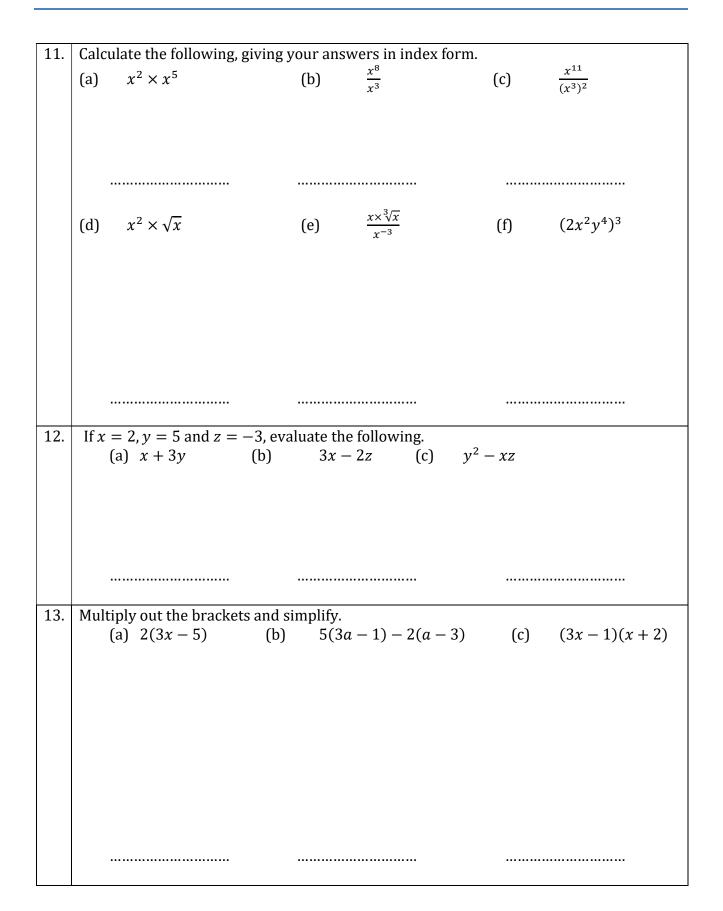
NUMBER

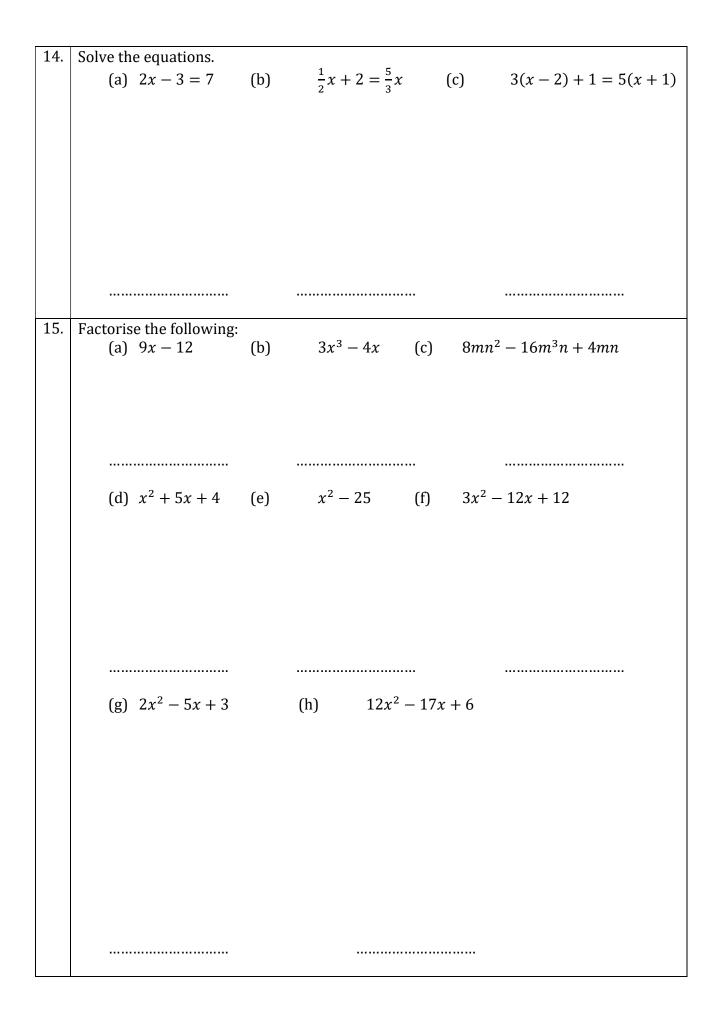


4.	Express $3\sqrt{8} + 2\sqrt{50}$ in the form $a\sqrt{b}$, where <i>a</i> and <i>b</i> are integers.
5.	
5.	Rationalise the denominators.
	(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{2}}$ (c) $\frac{14+3\sqrt{7}}{\sqrt{7}}$
6	
6.	Express the recurring decimals below as fractions in their lowest terms. (a) 0.72 (b) 0.34
1	2
7.	Show that $\left(\frac{1}{8}\right)^{-\frac{2}{3}} = 4.$

8.	A man drives 115 miles in 2 hours 30 minutes.
	Calculate his average speed in miles per hour.
9.	Simplify $\frac{2^3 \times 2^5}{8^2}$, giving your answer as a power of 2. You must show your working.
	$\frac{1}{8^2}$, giving your answer as a power of 2. Fou must show your working.
10.	Show that $1 + 1 + 5 = 13$
	Show that $\frac{1}{4} + \frac{1}{3} \div \frac{5}{6} = \frac{13}{20}$

ALGEBRA





16.	Make <i>x</i> the subject of the equation	s helow	
10.	(a) $y = 3x - 2$		2x + y = 3(x - z)
	(c) $3z = \frac{x^2}{2} - 1$	(d)	$y = \frac{3x-2}{5-x}$
	$(c) 32 = \frac{1}{2}$	(u)	$y = \frac{1}{5-x}$
17.	Labiala of a number multiple it by 2 on		
	x representing the number I thought o		Iy answer is 17. Form an equation with it to find my number. Any answers
	without sufficient working will gain no		it to find my number. Any answers
	without sufficient working win gain no	, marks.	
18.	Solve the equation $x^2 - 5x - 14 = 0$		

19.	Solve the simultaneous equations:	3x - 2y = 10 $x + y = 5$	
20.	Solve the simultaneous equations:	$\begin{aligned} x + y &= 3\\ x^2 - 3y &= 1 \end{aligned}$	
21.	Solve the equation $3^{3x+1} = 9^{2x-1}$		

1					
23.	Solve the inequality $3x + 1 \le 13$				
24.	Solve the inequality $x^2 - 4x + 3$	≤ 0			
25.	Simplify the following. (a) $\frac{x}{2} - \frac{x}{3}$ (b)	$\frac{3x-1}{3} + \frac{x+2}{5}$	(c)	$\frac{1}{x} + \frac{3}{2x}$	

	(d) $\frac{x+1}{x^2+4x+3}$	(e)	$\frac{3}{x-1} + \frac{4}{x+2}$	(f)	$\frac{4}{x^2+2x-3} + \frac{1}{x+3}$
26.	Solve (a) $x = \frac{15}{x} - 2$	(b)	$\frac{2x-1}{x+3} = \frac{x}{2x+2}$	(c)	$\frac{x}{x-1} + \frac{1}{x} = \frac{11}{6}$
27.	Express $x^2 + 6x + 4$ in 6	complete	ed square form.		

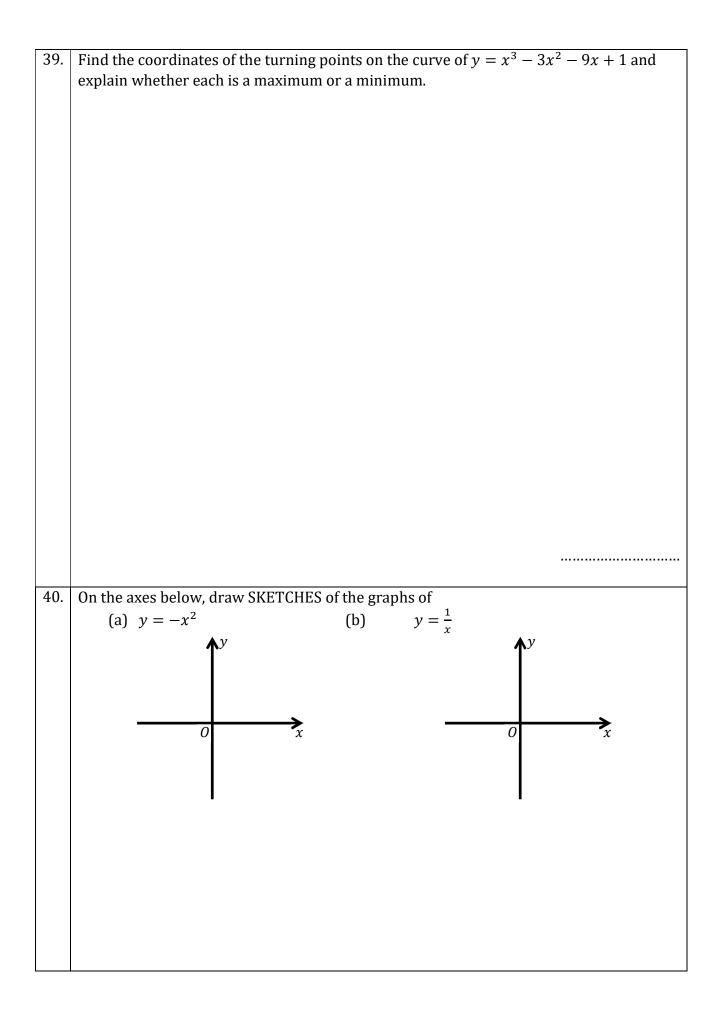
28.	Rewrite $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$
20.	Rewrite $3x = 0x + 2$ in the form $a(x + b) + c$
29.	By using the discriminant, find how many solutions there are to the quadratic equation
47.	
	$3x^2 + 2x + 3 = 0$. Do not solve the equation.

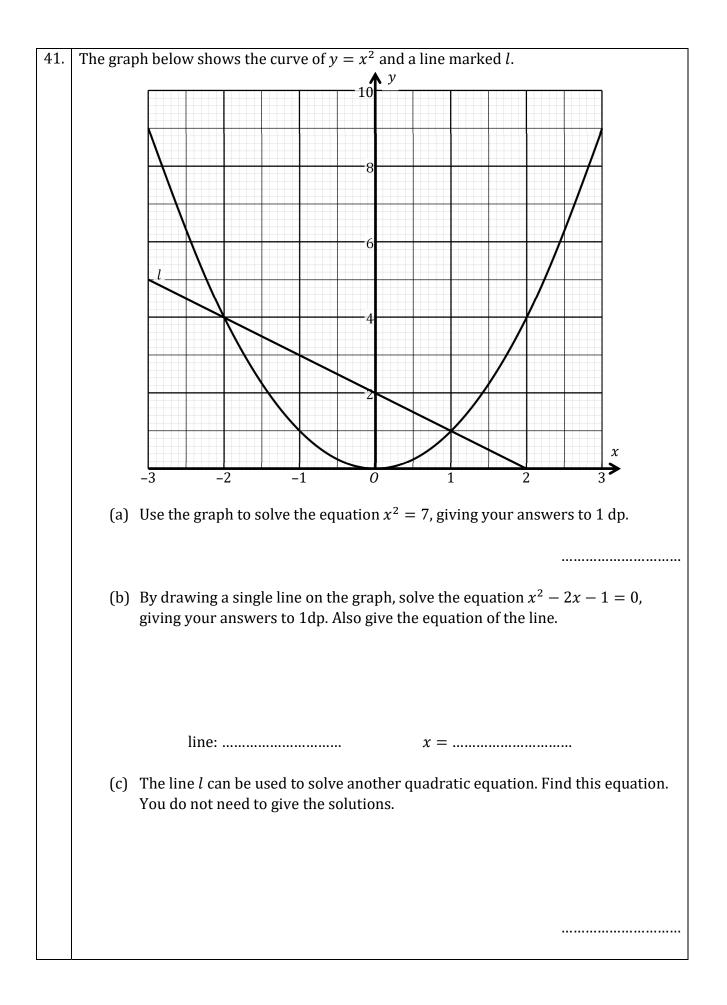
SEQUENCES, FUNCTIONS & GRAPHS

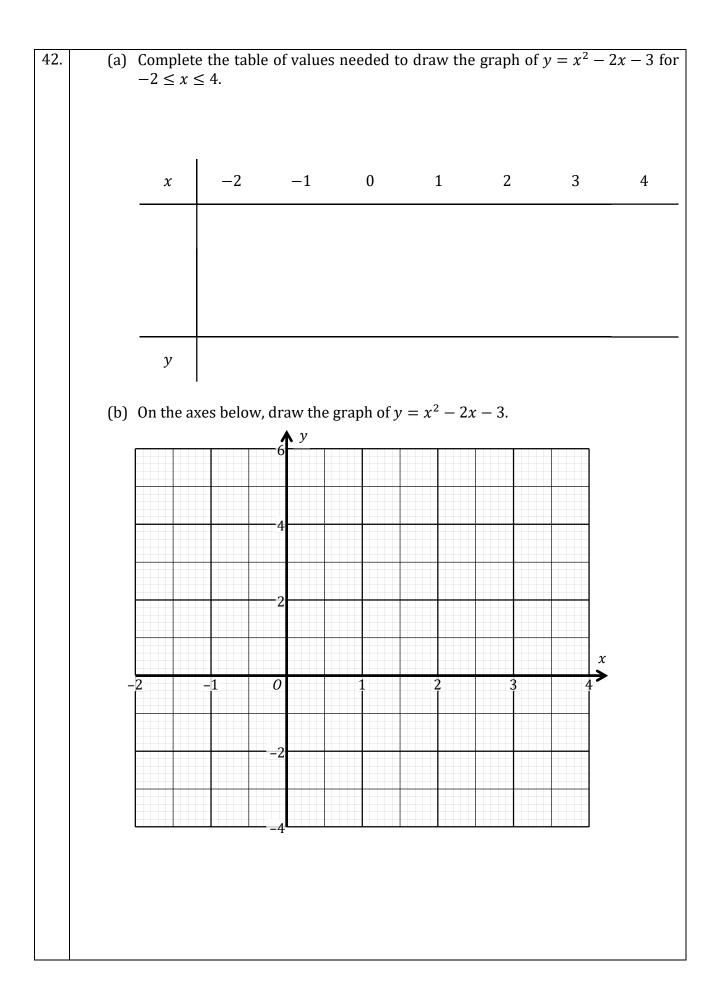
Find the next three terms and give a rule for the n^{th} term in each of the sequences below. 30. (b) (a) 4, 7, 10, 13, 16, ... 2, 7, 12, 17, ... 14, 12, 10, 8, ... (c) n^{th} term = n^{th} term = n^{th} term = $\frac{1}{4}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}, \frac{5}{8}, \dots$ (d) 1, 4, 9, 16, 25, ... (e) 3, 4, 7, 11, 18, ... (f) n^{th} term = n^{th} term = $n^{\rm th}$ term = 31. Write down the values of the gradient and the intercept for the lines representing the equations. 2x + 5y = 10(a) y = 3x + 1(b) gradient =gradient =intercept = intercept = $f(x): x \mapsto 3x - 1$ 32. Find: (a) f(5) (b) x such that f(x) = 20..... (c) f(2x+3)(d) the inverse of f(x).

33.	If $f(x) = \frac{2x-1}{3-x}$, find the inverse function $f^{-1}(x)$.
34.	$g(x) = x^2$ and $h(x) = 3x - 2$. Find (a) $gh(3)$ (b) $hg(3)$ (c) $gg(3)$
	$(a) gin(3) \qquad (b) \qquad iig(3) \qquad (c) \qquad gg(3)$
	(d) x such that $gh(x) = 16$ (e) x such that $g(x) = h(x)$ (f) $hh(x)$
35.	Find the equation of the line which passes through the point with coordinates (2,11) and
	is parallel to the line $y = 3x + 1$.

36.	The line <i>l</i> connects the points with coordinates (2, 3) and (10, 7).					
	(a) Find the exact length of l (b) Find the coordinates of the midpoint of l					
	······					
	(c) Find the equation of the line <i>l</i>					
37.	Describe which value or values, if any, must be excluded from the domains of the					
	functions below.					
	(a) $f(x) = \frac{1}{x-1}$ (b) $g(x) = \sqrt{x-5}$ (c) $h(x) = \frac{1}{\sqrt{3-x}}$					
	x-1 V3-x					
38.	$\sum_{i=1}^{n} dy$ for the equation of all $i = 1$					
50.	Find $\frac{dy}{dx}$ for the equations below.					
	(a) $y = x^3 - 3x^2 + 4x + 1$ (b) $y = \frac{1}{x^2}$					
	2 —					
	(c) $y = \frac{3}{2x^3} - 4\sqrt{x}$ (d) $y = (3x - 1)(2x + 3)$					
	······					







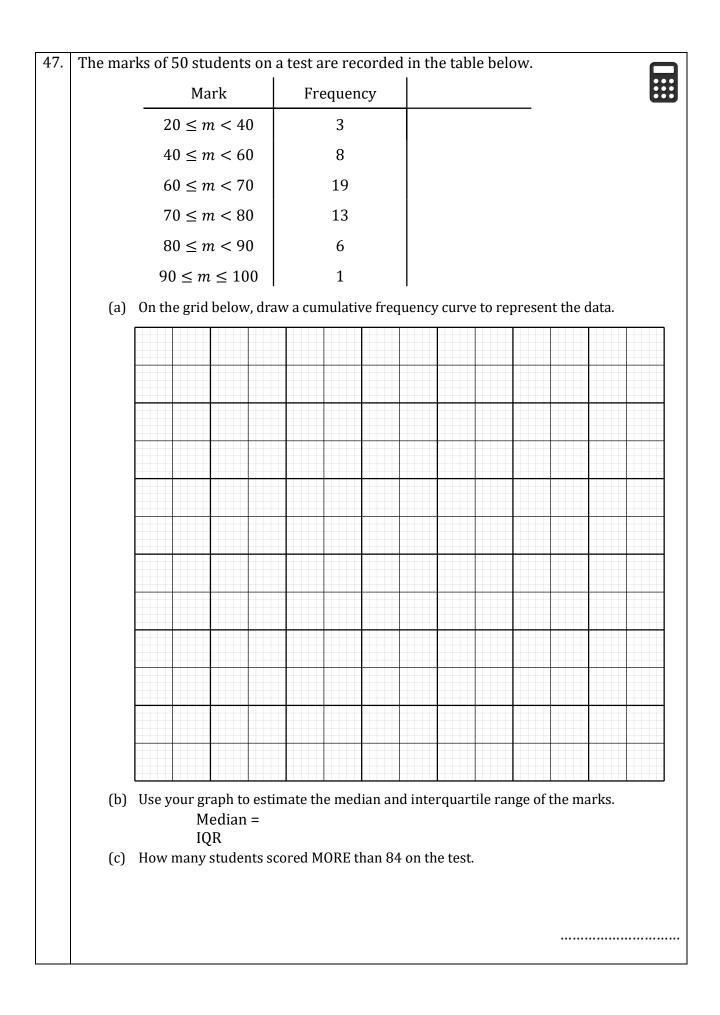
(C)	By drawing a suitable tangent, estimate the gradient of the graph when $x = 2$.
Ċ	
(d)	By differentiating the original equation, find the exact gradient when $x = -1$.
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(d)	
(d)	By differentiating the original equation, find the exact gradient when $x = -1$.

GEOMETRY

43.	Find the length of side <i>AC</i> in the triangle shown alongside. Give your answer to 3 significant figures.
44.	The triangle <i>ABC</i> is shown in the diagram alongside.
	 (a) Find the size of angle ABC. Give your answer to 1 decimal place. 8cm 4cm 4cm 6cm 8
	······································

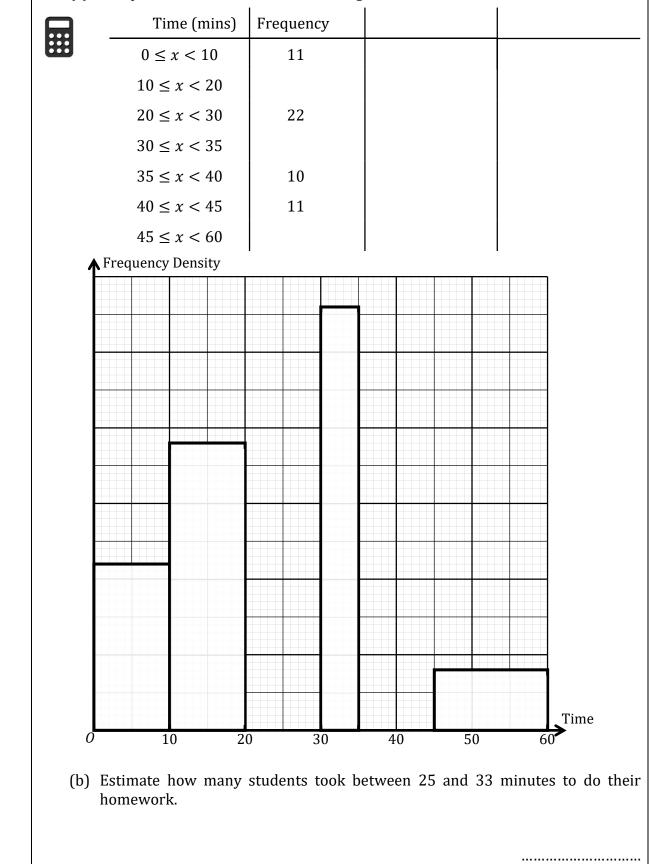
STATISTICS

45.	passes t	bability that Gunthe he test is 0.7, but if h Draw a tree diagran	e does not revise, th		es, the probability he sses is 0.4.	
	(b) Find the probability Gunther revises and passes the test.					
	(c)	Find the probability	Gunther fails the te	est.		
46.		le below shows the r	narks obtained by 2	25 students in a tes	st. Estimate the mean	
	mark.	Mark m	Frequency <i>f</i>			
		$50 \le m < 60$	2			
		$60 \le m < 70$	9			
		$70 \le m < 80$	7			
		$80 \le m < 90$	4			
		$90 \le m \le 100$	3			
			-			



48. The students in a school measure the time they take to complete their homework one night. The results are shown in the table and histogram below.

(a) Complete both the table and the histogram.



For your maths A Level, you are required to be familiar with using a Large Data set to carry out calculations, statistical analysis and drawing conclusions in context. The data set that you need to be familiar with is based on data from 8 weather stations across the world. Below is an extract from the information page on the large data set. Using this and your own general knowledge answer the questions below. You may use the internet to research the answers to the questions you do not know.

The data set consists of weather data samples provided by the Met Office for five UK weather stations and three overseas weather stations in the time periods May to October 1987 and May to October 2015. The weather stations are labelled on the maps shown:

• in the UK - Camborne, Heathrow, Hurn, Leeming and Leuchars

• overseas - Beijing, Jacksonville and Perth

Further information around our data source can be accessed at http://www.metoffice.gov.uk/

The Met Office provides data for a number of different weather variables. Our data set includes data for eleven variables recorded across the weather stations during the set periods of time:

Daily Mean Temperature

Air temperatures are recorded by thermometers in a louvered screen 1.25 metres above short grass, except at some Weather Centre's and Climate Data Logger stations, where observations are made from a non-standard roof top exposure.

Values are noted in degrees and tenths (Degrees Celsius) and values below 0 Deg C are preceded by a minus sign. A reading which is not available is listed as 'n/a'.

The daily mean air temperature (0900-0900 GMT) is the average of the hourly temperature readings during this period.

Daily Total Rainfall

Daily totals refer to 24 hour periods commencing at 0900 Greenwich Mean Time (1000 British Summer Time) on the day of entry and includes any solid precipitation, such as snow or hail, which is melted and measured in the same way as rainfall.

A 'trace or tr' of rain is an amount less than 0.05mm. A reading which is not available will be shown by 'n/a'. All totals are in millimetres (mm).

Daily Total Sunshine

At most Met Office stations sunshine is measured by an instrument that measures the amount of solar radiation exceeding a threshold.

Sunshine amounts are recorded in hours and tenths and show the amount of bright sunshine recorded on the day of entry. A reading which is not available is listed as 'n/a'.

Daily Maximum Relative Humidity

The relative humidity is a measure of how close the air is to being saturated with water vapour. Relative humidities of above 95% are associated with mist and fog. A reading which is not available is listed as 'n/a'.

Daily Mean Windspeed, Daily Maximum Gust, Daily Mean Wind Direction and Daily Maximum Gust Direction

• Wind speeds are given in knots (1 knot = 1.15mph).

The daily mean speed is averaged over the 24 hours from 0000GMT on the date given.

• The maximum gust speed is the maximum instantaneous speed that occurred during the 24

hours from 0000GMT on the date given. Readings for both variables which are not available are listed as 'n/a'.

• The daily mean wind direction is averaged over the 24 hours from 0000GMT on the date given, rounded to the nearest 10 degrees

• The direction of the maximum gust is that direction from which the wind was blowing when the maximum gust during the hour commencing at the time of entry occurred, and is measured in degrees from true north.

For all locations, the speed of the wind is also categorised according to the Beaufort scale. For the UK locations, the wind and gust direction measurements are also listed as cardinal directions.

Cloud cover

Cloud cover is measured in eighths (Oktas).

Cloud cover is the fraction of the celestial dome covered by cloud.

Visibility

Visibility is defined as the greatest distance at which an object can be seen and recognized in daylight, or at night could be seen and recognized if the general illumination were raised to daylight level. It is measured using visiometer at automatic sites but used to be done by observers at manual stations except at some Weather Centres and Climate Data Logger stations, where observations are made from a non-standard roof top exposure.

Visibility is measured horizontally.

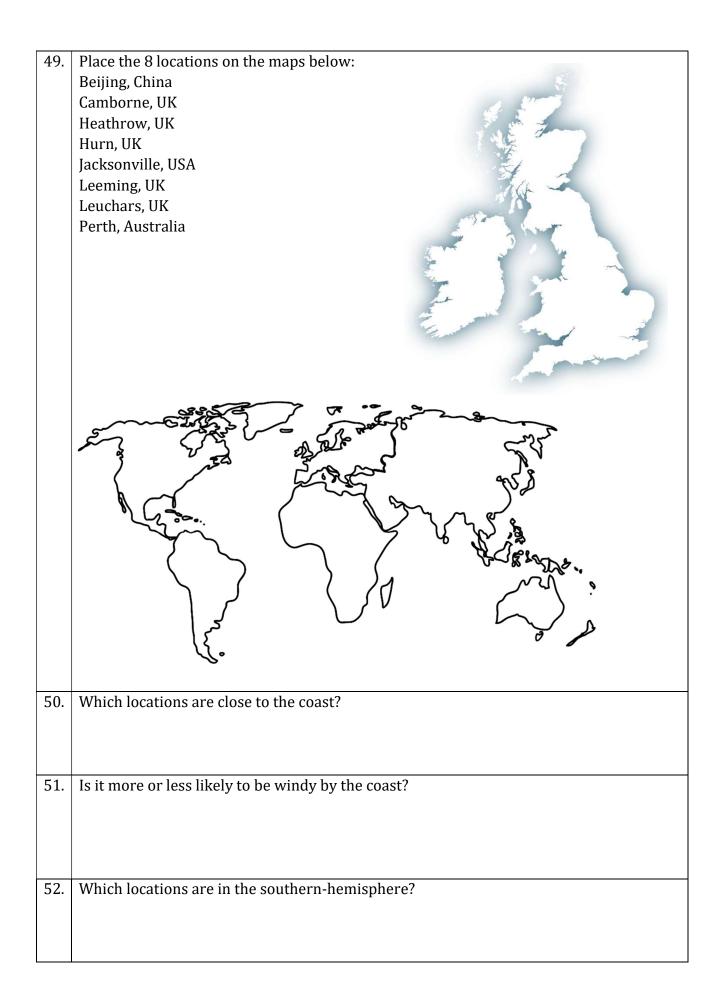
Values are noted in decametres (Dm)

A dash indicates data not available.

Pressure

• The mean sea level pressure has been calculated from a measurement made at station level.

• Units - the pressure unit used in meteorology was previously the millibar (one bar = 1000 millibars). However, this has been replaced by the SI unit of pressure — the pascal (Pa). One hectopascal (hPa) = 1 millibar (mb).



53.	Will these locations in the southern-hemisphere have higher temperatures in July or
	December?
54.	Which locations are close to the equator?
55.	What can us infor about the temperature and amount of reinfall about locations along to
55.	What can we infer about the temperature and amount of rainfall about locations close to
	the equator?
F (
56.	Which locations will generally have the highest temperatures?
57.	Which locations will generally have the lowest temperatures?
58.	Which locations will generally have the most rainfall?
59.	Which locations will generally have the least rainfall?
60.	Which locations are generally the windiest?
61.	Which locations are generally the least windy?
62.	What does "tr" mean?
	mataoos a mount
63.	What does "n/a" mean in the context of the data?
03.	what uses ha mean mule concert of the data?

64.	In what years do we have data about?
65.	Which months have we been given data for?
66.	Which months have we not been given any data for?
67.	How reliable will any predictions we make about the daily mean wind speed in January? Why?
68.	What units are used to measure wind speed?
69.	What scale is used to measure wind speed?
70.	What units are used to measure cloud cover?
71.	What units are used to measure visibility?
72.	What units are used to measure pressure?

You should have been able to cope with the questions so far with relative ease. These questions were based on the IGCSE syllabus. The remainder of the workbook is based on the syllabus for the Further Maths GCSE.

You are likely to cover most of this work again at the start of the Sixth Form, but it is important that you give yourself as firm a foundation as possible. The best way to do this is by giving yourself a reminder of the more complicated topics you have seen, and this is the aim of the rest of this workbook. If you complete the remainder of the workbook you can be confident that you will cope with the beginning of the A Level course.

As with the first part of the workbook, you must show your working. The only questions you may use a calculator on are marked with an image of a calculator.



If you need any help on any questions you should first go to <u>www.hegartymaths.com</u> and use the videos online to help you. Failing that, you can email <u>gjs@reigategrammar.org</u> and you should get a reply within a few days.

ALGEBRA

73.	The quadratic equation $4x^2 + 12x + k = 0$ has one repeated solution.	
	Find the value of <i>k</i> .	
74.	Rationalise the denominator in each of the fractions given.	
	(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{2+\sqrt{3}}{5-\sqrt{3}}$	
	$\sqrt{2}$ $\sqrt{2}$ $\sqrt{3}$	
75.	Given that $f(x) = x^3 - 3x^2 + 4$,	
/ 0.	(a) show that $(x - 2)$ is a factor of $f(x)$	
	(h) fully for the size f(w)	
	(b) fully factorise $f(x)$	

	(c) solve $f(x) = 0$
	(d) write down the coordinates of one of the turning points on the graph of $y = f(x)$.
76.	For what value of k is $(x + 1)$ a factor of $x^4 - 2x^2 + k$?
/ 01	
77.	Sketch the graph of $y = x^2 - 4x - 5$, giving the coordinates of the turning point(s) and
	any points where the graph intersects the coordinate axes.
	$\mathbf{\uparrow} \mathbf{v}$
	<i>0 x</i>
1	

COORDINATE GEOMETRY

78.	A and B	are the points $(3, -2)$ and $(9, 6)$. Find:	
		the midpoint of the line <i>AB</i> ,	
	(1)		
	(b)	the gradient of the line <i>AB</i> .	
	(c)	the length of the line <i>AB</i> .	
79.		<i>l</i> passes through the points $D(-2,1)$ and $E(6,5)$.	
	(a)	Find the equation of the line <i>l</i> .	
	(b)	Find the equation of the line parallel to <i>l</i> , which passes through	the point $F(3,4)$.
	(c)	Find the equation of the perpendicular bisector of the line <i>DE</i> .	
	(-)		

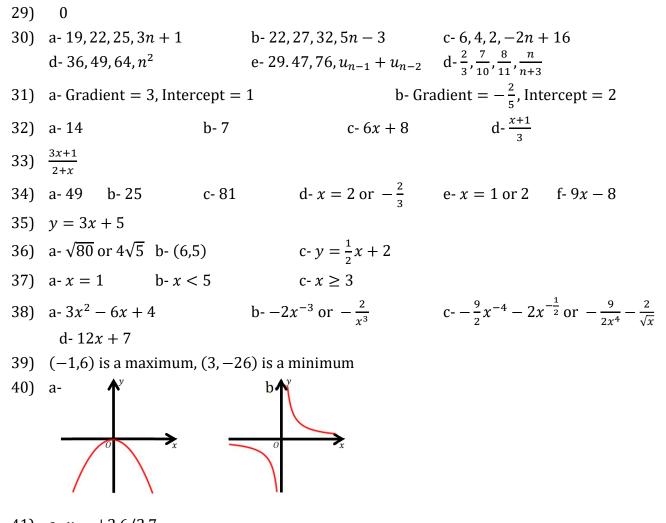
80.	The points $A(0,5)$, $B(4,1)$ and $C(2,7)$ form a triangle. Show that the triangle is right-
	angled and state which angle is the right angle.
81.	The line <i>l</i> passes through the points (2,9) and (5,4). Find the equation of <i>l</i> , giving your answer in the form $ax + by + c = 0$ where <i>a</i> , <i>b</i> and <i>c</i> are integers.
82.	Find the centre and radius of the circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$
	Centre = Radius =

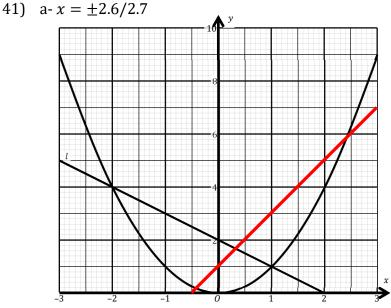
83.	The circle <i>C</i> has equation $(x - 1)^2 + (y + 2)^2 = 25$.	
	(a) Determine whether the point $A(-1,2)$ lies inside, on, or outside C.	
	(b) Chow that the point $P(A, 2)$ lies on C	
	(b) Show that the point B (4,2) lies on C .	
	(c) Find the equation of the tangent to the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B , giving your answer in the circle C at B .	he
	form $ax + by + c = 0$ where a, b and c are integers.	

CALCULUS

84.	The curve $y = x^3 - 6x^2 - 15x + 2$ has a turning point at (-1,10). Determine whether it is a maximum or a minimum point.
85.	A particle moves in a straight line such that at time t its displacement, s, from a fixed point O on that line is given by $s = 2t^3 - 3t^2 + 4$. Find: (a) expressions for the velocity and acceleration in terms of t
	$v = \dots$ $a = \dots$
	(b) the times when it is at rest
	(c) how far it is from <i>O</i> when it is at rest
	(d) the initial acceleration of the particle.

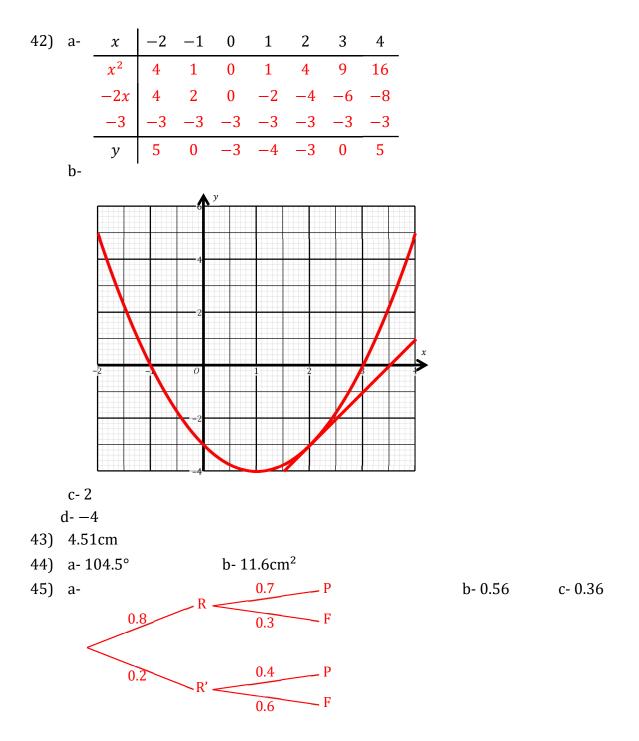
1) $a - \frac{1}{4}$ b-1 $c - \frac{1}{8}$ d-5 e-27 f- $\frac{1}{4}$ 2) $3\sqrt{2}$ 3) a- $8\sqrt{3}$ b- 45 $c - 1 + \sqrt{3}$ 4) $16\sqrt{2}$ 5) $a - \frac{\sqrt{3}}{2}$ b- $2\sqrt{2}$ c- $2\sqrt{7}$ + 3 $b - \frac{31}{90}$ 6) $a - \frac{8}{11}$ 7) $\left(\frac{1}{8}\right)^{-\frac{2}{3}} = 8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ 8) 46 9) 2² 10) $\frac{1}{4} + \frac{1}{3} \div \frac{5}{6} = \frac{1}{4} + \frac{1}{3} \times \frac{6}{5} = \frac{1}{4} + \frac{1}{1} \times \frac{2}{5} = \frac{1}{4} + \frac{2}{5} = \frac{5+8}{20} = \frac{13}{20}$ 11) a- x^7 b- x^5 c- x^5 d- $x^{2.5}$ e- $x^{4\frac{1}{3}}$ or $x^{\frac{13}{3}}$ f- $8x^6v^{12}$ 12) a-17 b- 12 c- 31 13) a - 6x - 10 b - 13a + 1 $c - 3x^2 + 5x - 2$ 14) a - x = 5 $b - x = \frac{12}{7}$ c-x = -515) a - 3(3x - 4) $b - x(3x^2 - 4)$ $c - 4mn(2n - 4m^2 + 1)$ $e - 3(x - 2)^2$ f - (x + 5)(x - 5) g - (2x - 3)(x - 1)d-(x + 1)(x + 4)h-(3x - 2)(4x - 3)16) $a - x = \frac{y+2}{3}$ b - x = y + 3z $c - x = \sqrt{2(3z+1)}$ $d - x = \frac{5y+2}{3+y}$ 17) 3x + 2 = 17, x = 518) x = -2 or 719) x = 4, y = 120) x = -5, y = 8 or x = 2, y = 121) x = 322) x = -3.45 or 1.45 23) $x \le 4$ 24) $1 \le x \le 3$ 25) $a - \frac{x}{6}$ $b - \frac{18x+1}{15}$ $e - \frac{7x+2}{(x-1)(x+2)}$ $f - \frac{1}{x-1}$ $C-\frac{5}{2x}$ $d - \frac{1}{r+3}$ 26) x = -5 or 3 $b - x = -\frac{2}{3} \text{ or } 1$ $c - x = \frac{2}{5} \text{ or } 3$ 27) $(x+3)^2 - 5$ 28) $3(x-1)^2 - 1$



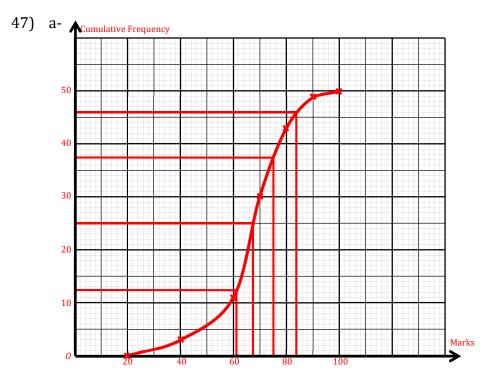


b-
$$y = 2x + 1, x = -0.4$$
 or 2.4

c-
$$x^2 + x - 2 = 0$$

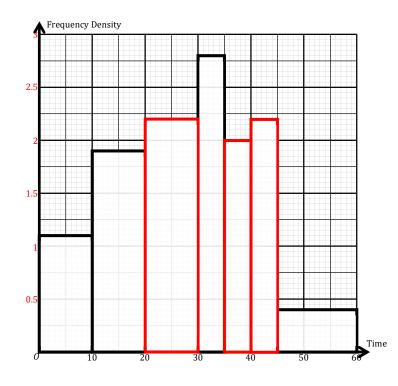


46) 73.8



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b- median = 67, IQR = 25
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c- 4 Class Width Time (mins) Frequency Frequency Density 48) a- $0 \leq x < 10$ 10 1.1 11 $10 \le x < 20$ 19 10 1.9 $20 \le x < 30$ 22 2.2 10 $30 \le x < 35$ 5 2.8 14 $35 \le x < 40$ 2 10 5 $40 \le x < 45$ 5 2.2 11 $45 \le x < 60$ 15 0.4 6



b- 19 or 20

73) 9 $a - \frac{3\sqrt{2}}{2}$ $b - \frac{7\sqrt{3} + 13}{22}$ 74) a- f(2) = $2^3 - 3 \times 2^2 + 4 = 0$, \therefore Factor b- $(x - 2)^2(x + 1)$ c- 2 or = -1 d- (2,0) 75) 76) 1 +ve quadratic curve, turning point at (2, -9), intersects axes at (-1, 0), (5, 0), (0, -5)77) $b-\frac{4}{3}$ a- (6,2) c- 10 78) a- $y = \frac{1}{2}x + 2$ b- $y = \frac{1}{2}x + \frac{5}{2}$ c- y = -2x + 779) Either Pythagoras' Theorem or two perpendicular sides 80) 5x + 3y - 37 = 081) Centre = (3,1), radius = 5 82) b- $(x-1)^2 + (y-2)^2 = 3^2 + 4^2 = 25$ c-3x + 4y - 20 = 0a- Inside 83) Maximum 84) a- $v = 6t^2 - 6t$, a = 12t - 685) b- t = 0 and t = 1

c- At t = 0, s = 4m, at t = 1, s = 3m d- -6ms⁻²